

**George Mason University**  
**College of Education and Human Development**  
Mathematics Education Leadership

MATH 610 (6M1) - Number Systems and Number Theory for K-8 Teachers

3 Credits, Fall 2016

Wednesdays 4:30 – 7:10

8/31: Frederick County School Board Office, Conference Room

9/07-12/14-James Wood Middle School

Frederick County Public Schools

1313 Amherst St. Winchester, VA 22601

**Faculty**

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**Prerequisites/Corequisites**

Admission to the Mathematics Education Leadership Master's Degree Program or instructor permission.

**University Catalog Course Description**

This course covers the topics: ways of representing numbers, relationships between numbers, number systems, the meanings of operations and how they relate to one another, and computation within the number system as a foundation for algebra. It also includes episodes in history and development of the number system, and will examine the developmental sequence and learning trajectory as children learn this material.

**Course Overview**

This course, for future K-8 mathematics teacher specialists, examines concepts contained in the number and operations strands of the Virginia Standards of Learning (SOL), Common Core State Standards (CCSS), and/or referenced in the National Council of Teachers of Mathematics (NCTM) Principles and Standards. Through a coordinated program of activities, participants will learn to explore the structure of number systems, properties of numbers and develop number sense, computation and estimation concepts and skills.

**Course Delivery Method**

Class sessions will engage participants in hands-on, in-depth inquiry of mathematics topics related to the areas of number systems and number theory..

## **Learner Outcomes or Objectives**

This course is designed to enable students to do the following:

- Use numerous representations and conceptual models
- Develop flexibility in problem solving
- Explain number concepts and interpret student work in many ways

## **Professional Standards**

(Specific content and learning outcomes will include the following standards from **NCTM CAEP Mathematics Content for Elementary Mathematics Specialist (Addendum to the NCTM CAEP Standards 2012)**):)

Upon completion of this course, students will have met the following professional standards:

C.1. Number and Operations -To be prepared to support the development of student mathematical proficiency, all elementary mathematics specialists should know the following topics related to number and operations with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

C.1.1 Counting and cardinality, comparing and ordering, understanding the structure of the base ten number system with particular attention to place value, order of magnitude, one-to-one correspondence, properties, and relationships in numbers and number systems – whole numbers, integers, rationals, irrationals, and reals

C.1.2 Arithmetic operations (addition, subtraction, multiplication, and division) including mental mathematics and standard and non-standard algorithms, interpretations, and representations of numbers – whole numbers, fractions, decimals, integers, rationals, irrationals, and reals

C.1.3 Fundamental ideas of number theory – divisors, factors and factorization, multiples, primes, composite numbers, greatest common factor, and least common multiple

C.1.4 Quantitative reasoning and relationships that include ratio, rate, proportion, and the use of units in problem situations

C.1.5 Historical development and perspectives of number, operations, number systems, and quantity including contributions of significant figures and diverse cultures

## **Required Texts**

--*PRINT*--

Fosnot, C.T., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division*. Portsmouth, NH: Heinemann.

Schifter, D., Bastable, V., & Russell, S. J. (2016). *Number and operations, part 1: Building a system of tens casebook*. Reston: National Council of Teachers of Mathematics.

--ONLINE--

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Retrieved from [http://www.nap.edu/catalog.php?record\\_id=9822](http://www.nap.edu/catalog.php?record_id=9822).  
(We will look at excerpts from this text.)

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Authors. [http://www.corestandards.org/wp-content/uploads/Math\\_Standards1.pdf](http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf)

Progressions Documents for the Common Core Math Standards. (2016, August 2016). Number and Operations in Base Ten, K–5 [Blog post]. Retrieved from [http://commoncoretools.me/wp-content/uploads/2015/03/ccss\\_progression\\_nbp\\_k5\\_2015\\_03\\_16.pdf](http://commoncoretools.me/wp-content/uploads/2015/03/ccss_progression_nbp_k5_2015_03_16.pdf) (Site may not be accessible through school servers.)

Progressions Documents for the Common Core Math Standards. (2016, August 2016). Counting and Cardinality; K–5, Operations and Algebraic Thinking [Blog post]. Retrieved from [https://commoncoretools.files.wordpress.com/2011/05/ccss\\_progression\\_cc\\_oa\\_k5\\_2011\\_05\\_302.pdf](https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf). (Site may not be accessible through school servers.)

Other readings as assigned.

### ***Recommended:***

National Council of Teachers of Mathematics. (2010). *Developing Essential Understanding of Number & Numeration Pre-K-Grade 2*. Reston, VA: Author

Van de Walle, J., Karp, K., & Bay-Williams, J. (2012). *Elementary and Middle School Mathematics: Teaching Developmentally*. (7<sup>th</sup> or 8<sup>th</sup> edition) Boston, MA.: Pearson Education.

## **Course Performance Evaluation**

Students are expected to submit all assignments on time in the manner outlined by the instructor (e.g., Blackboard, Tk20, hard copy). If you anticipate being late with an assignment, you must advise me before the due date.

- **Assignments and Examinations**

1. Reflection Journal (This is a Performance Based Assessment.)

You will write a short (one to one and half-page) reflection in response to a prompt given at the end of class. These reflections are due before the next class and to be posted to Blackboard on the discussion board. For each week a reflection is due you should respond to at least one post made by your colleagues. For best results, posts should be made earlier in the week to allow for responses from colleagues. Grading will be limited to verifying that responses are appropriate in topic and scope. There are no 'right' answers. Please use this time and space to give your honest answers, not ones you feel are expected. Your journal entries will serve to document your preparation for class (and your growth over time). There are 5 journal entries over the course of this class, including one specifically related to the historical significance of mathematics (Mathematics Historical Context Reflection). **Each journal entry is 2 points**

## 2. Quizzes

At the quarter and three quarter points of the semester, you will take a short quiz covering the material covered up to the point of the quiz. These quizzes are designed to provide you with an idea of the types of questions to anticipate on the midterm and final exam and to provide you with feedback as to your understanding of the material covered in class. There are 2 quizzes over the course of this class. **Each quiz is 5 points.**

## 2. Attendance and class participation

Class participation grades will be based on informed, relevant, productive, and respectful contributions (questions as well as comments and responses) to class discussions; attendance will be a factor in this part of the grade. Included in this grade is punctual completion of all homework assignments.

## 3. Case study

During the course we will read and discuss in class case studies and pedagogical strategies found in the classroom. For this assignment, you are to write a short (~1,000 word) case study describing a mathematical discussion involving one or more students, similar to these cases. A case is neither a complete transcript of a lesson nor as prefabricated as an interview, although it is very helpful to include direct quotes and dialogue from students. You must base your case on a conversation for which you were present, and preferably in which you were involved, but it could come out of a lesson you observed, or a conversation among two or more students. You may choose to narrow in on one or two students, or on one small group, or you may describe a whole-class conversation. The most important thing is that the episode illustrates some aspect of children's mathematical thinking. It must also center on a mathematical topic involving children's thinking about number and number sense.

In writing your case study, begin by describing briefly the class's larger context (including grade level) and the mathematical topic; then describe the relevant parts of the conversation in as much detail as you can manage. Include what you are thinking as you work with the students. Finish up by summarizing your evaluation of the students involved and saying what issues and questions you still have after this conversation.

Include an analysis of the students' thinking, and questions the case raises for you. It is important that your reflection address teaching issues beyond the one topic and set of students involved. This will document your ability as a reflective practitioner to make connections that inform your teaching practice more broadly. Please use 1 inch margins, Times New Roman 12pt font and double spaced. Post this assignment to Tk2. Refer to the rubric posted on blackboard when completing this assignment.

4. Cognitively Demanding Task and Mock Coaching Session (This is a Performance Based Assessment.)

In this course we will study the teaching and learning of ideas related to number and number sense in K-8 mathematics and how to encourage a productive disposition in both our students and ourselves. As a summative evaluation of the pedagogical aspects of this course, you will develop a lesson centered around a task that fosters learning these concepts, teach and document the lesson, confer with a colleague in a mock-coaching session, and give a short (15-minute) presentation to the class on the process. The lesson draft checkpoint includes items a-d below. The final lesson paper you submit must include all of the following components:

a) **The Task**

Select or develop a cognitively demanding task intended for use with the students you teach, which encourages student development of number and number sense structure. You may use or adapt a problem from class materials, but be sure it is appropriate for the target audience. You may select your own class as the target audience or serve as a guest teacher if you are not currently a math teacher.

1. Write a paragraph explaining what essential number and number sense understanding is addressed in this problem and your specific learning goal for the lesson.
2. Specify prerequisite knowledge for accessing the task, but also explain how the task is accessible to everyone in your class.
3. Solve the problem as many ways as you can, and during class work with a group to compile a collection of students' possible solution strategies.
4. Describe what materials will be available to students and why you are offering them. Describe the directions and clarifications that you will give to the class in addition to the written directions.
5. Make a list of the problem solving strategies and approaches you anticipate seeing in the classroom as you teach this lesson. Discuss the meaning of student approaches to the task and how this understanding impacts the learning goal for the lesson.
6. Identify which strategies you will select for use in your classroom discussion in order to further your mathematical goal. Present the strategies in a logical sequence, which you should explain, including the sequence rationale. Lastly, explain how the strategies connect to one another, to prior knowledge or to future learning.
7. As a group you will meet with the instructor to discuss progress. (All of these items should be included in your final paper.)

- b) **Lesson Plan** – In your lesson plan include all of the above using the provided template, which also includes other key elements of a lesson plan. You may work in a group to create a cluster of related tasks.
- c) **Mock Coaching Session** – Before teaching the lesson, you and a partner (or two partners) will participate in a mock coaching session during class. You will be paired with another member of the class who teaches a grade different than yours. You will take turns acting as the coach and the coachee using a guide that assists with a coaching conversation. Your goal is to practice engaging teachers in conversations about the quality and scope of their lesson plans and encouraging them to think deeper about student understanding across grade levels.  
Write at least a one-page reflection on the Mock Coaching session, describing your actions and reactions both as a coach and as a coachee.
- d) **Teach** – Each individual will teach their lesson to students and write a two-page (1” margins, Times New Roman 12pt font and double spaced) reflection on how their lesson went, including what strategies students used to approach the problem, what ideas were raised in its discussion, and to what extent your students' understanding of the underlying number/number sense concepts or ability to apply them changed as a result of the lesson. Be specific.
- e) **Presentation Handout** – Create a one-page handout (you may use front and back if necessary, but it must fit on one sheet) summarizing your lesson for the class. Include the problem, grade level(s), mathematical topics addressed, how the cluster of tasks is related (if applicable) and anything your colleagues would need to know in order to use the lesson, including (briefly) any difficulties the students tended to encounter. You should include pictures of student work that serve as evidence of student thinking. The handout should not be the same as your lesson plan (just select details!), and will be turned in with the main paper. On the day of your group’s presentation bring a copy for each class member and for the instructor.
- f) **Due Dates** –  
 September 22 – A preliminary draft of the selected task is due (not yet taught).  
 October 13 – Mock Coaching Session. Lesson should be complete(a & b)  
 November 17 – Lesson should be complete and taught before this date (a-d)  
 December 1 – All documentation, including the handout is due (post on Blackboard under the assignments tab)  
 December 1 and 8 – Brief (10-minute) presentation to the class.

## 5. Final Exam

The course will conclude with a final exam addressing the concepts listed in the NCTM Standards for Elementary Mathematics Specialists.

- **Other Requirements**

This course is highly interactive. Successful completion of the course requires regular attendance, participation, and punctual completion of all class activities, homework and other activities. Satisfactory assignments are mathematically accurate, reflect the intent of the national and state standards, and demonstrate a comprehensive level of mathematical thinking and reasoning that shows mastery of the concepts being learned throughout the course.

This course will be taught using a variety of methods, including lecture/discussion, collaborative groups, and other formats appropriate to the content and goals of the course. Course instruction will model teaching methods that teachers can take back to their classrooms. There will be explicit discussion of the pedagogical approaches used in the course.

- **Course Performance Evaluation Weighting**

This 3 graduate credit hour course is dependent on both class discussion and activities to meet the objectives contained in the course description. Successful completion of the course requires regular attendance, participation, and punctual completion of all class activities, homework, and other assignments. Your grade for the course will be determined by seven items:

1) Journal Reflections	10%
2) Quizzes	10%
3) Case Study	15%
4) Cognitively Demanding Task and Mock Coaching Session (Lesson Plan)	15%
5) Midterm	20%
6) Final exam	20%
7) Attendance and Class participation	<u>10%</u>
	100%

- **TK20/Performance-Based Assessment(s) Submission Requirement**

Every student registered for any Mathematics Education Leadership course with a required TK20 performance-based assessment must submit these assessments (MATH 610: Reflection Journal Entries and Lesson Plan) to Tk20 through 'Assessments' in Blackboard. Failure to submit the assessments to Tk20 (through Blackboard) will result in the course instructor reporting the course grade as Incomplete (IN). Unless this grade is changed upon completion of the required Tk20 submission, the IN will convert to an F nine weeks into the following semester.

- **Grading Policies**

The course will be graded on a 100 point system:

100-98 A+    97-90 A    89-87 B+    86-80 B    79-70 C    69 and lower F

**Professional Dispositions**

Students are expected to exhibit professional behaviors and dispositions at all times.

**Core Values Commitment**

The College of Education and Human Development is committed to collaboration, ethical leadership, innovation, research-based practice, and social justice. Students are expected to adhere to these principles: <http://cehd.gmu.edu/values/>.

**GMU Policies and Resources for Students**

*Policies*

- Students must adhere to the guidelines of the Mason Honor Code (see

<http://oai.gmu.edu/the-mason-honor-code/>).

- Students must follow the university policy for Responsible Use of Computing (see <http://universitypolicy.gmu.edu/policies/responsible-use-of-computing/>).
- Students are responsible for the content of university communications sent to their Mason email account and are required to activate their account and check it regularly. All communication from the university, college, school, and program will be sent to students **solely** through their Mason email account.
- Students with disabilities who seek accommodations in a course must be registered with George Mason University Disability Services. Approved accommodations will begin at the time the written letter from Disability Services is received by the instructor (see <http://ods.gmu.edu/>).
- Students must follow the university policy stating that all sound emitting devices shall be silenced during class unless otherwise authorized by the instructor.

### *Campus Resources*

- Support for submission of assignments to Tk20 should be directed to [tk20help@gmu.edu](mailto:tk20help@gmu.edu) or <https://cehd.gmu.edu/api/tk20>. Questions or concerns regarding use of Blackboard should be directed to <http://coursessupport.gmu.edu/>.
- The George Mason University Writing Center staff provides a variety of resources and services (e.g., tutoring, workshops, writing guides, handbooks) intended to support students as they work to construct and share knowledge through writing (see <http://writingcenter.gmu.edu/>).
- The George Mason University Counseling and Psychological Services (CAPS) staff consists of professional counseling and clinical psychologists, social workers, and counselors who offer a wide range of services (e.g., individual and group counseling, workshops and outreach programs) to enhance students' personal experience and academic performance (see <http://caps.gmu.edu/>).
- The George Mason University Office of Student Support staff helps students negotiate life situations by connecting them with appropriate campus and off-campus resources. Students in need of these services may contact the office by phone (703-993-5376). Concerned students, faculty and staff may also make a referral to express concern for the safety or well-being of a Mason student or the community by going to <http://studentsupport.gmu.edu/>, and the OSS staff will follow up with the student.

**For additional information on the College of Education and Human Development, please visit our website <https://cehd.gmu.edu/>.**

**Class Schedule**

## MATH 610 • FALL 2016 • ASSIGNMENTS

	<b>Date/ Loc</b>	<b>Topic</b>	<b>Readings</b>	<b>Journal Entries/ Assignments</b> Due at the beginning of class on the given date.
1	8/31 FCPS SBO Conferenc e Room	Historical Number Systems	XXX	xxx
2	9/07 James Wood Middle School Rm. 106	Counting is more than 1,2,3	<i>Number sense on the horizon</i> Ch 3 in Constructing Number Sense, Addition, and Subtraction (Fosnot & Dolk)  <i>The Mathematical Knowledge Children Bring to School in Adding it Up – Ch 5</i>	HW: Exploring Elementary Mathematics (Weisglass), Read pp. 14-23; Do pp. 26-27, #1, 2, 6, 7, 8, 10, 11, 12, 15, 17
3	9/14 James Wood Middle School Rm. 106	Positional Number Systems	<i>“Mathematics” or “Mathematizing”</i> Ch 1 in Constructing Multiplication and Division (Fosnot & Dolk)  <i>The Base Ten Structure of Numbers</i> (Ch 2 in BST)	<b>Journal #1:</b> Exploring a Variety of Numbering Systems  <b>Case Study Checkpoint:</b> Email me with information about where you plan to observe student thinking.
4	9/21 James Wood Middle School Rm. 106	The Power of 10 and Problem Situations	Children’s Mathematics: Cognitively Guided Instruction, Ch 2  <i>The Strands of Mathematical Proficiency</i> (Adding it Up – Ch 4)  Review <i>Developing Whole Number Place Value Concepts</i> , Elementary and Middle School Mathematics: Teaching Developmentally, 7 <sup>th</sup> edition, pp. 187 – 200	<b>Journal #2:</b> Counting  <b>QUIZ #1</b> Pirate Math (Bases)  Focus Question from Ch 2 in BST  <b>Task-Based Lesson Checkpoint:</b> Part a (The Task) should be completed, at least through #5. You will have time in class to work on #5 – #7 in teams)  Abacus Homework
5	9/28 James Wood Middle School Rm. 106	Addition and Subtraction Strategies	<i>Making Sense of Addition and Subtraction Algorithms</i> (Ch 3 in BST)  <i>Variations in Both Addends</i>	<b>Journal #3:</b> Bases, Numeration, and Tools to Structure Base 10  CM: CGI, pp.15 -16, #2, 3

			<i>Unknown Problems</i>	
6	10/05 James Wood Middle School Rm. 106			<b>MIDTERM</b>
7	10/12 James Wood Middle School Rm. 106	Constructing Multiplication and Division Ideas	<p><i>Developing Multiplication Strategies and Big Ideas</i> Ch 3 in Constructing Multiplication and Division (Fosnot &amp; Dolk)</p> <p><i>Multiplication and Division: Sense Making and Meaning</i> (Kouba and Franklin), pp. 103-126.</p> <p>Common Core State Standards, p. 88 – 89</p>	<p>Focus Question from Ch 3 in BST</p> <p>Bring a reference copy of the table on page 89 of the CCSS Standards.</p>
8	10/19 James Wood Middle School Rm. 106	Expanding Multiplication	<i>Multiplication of Multidigit Numbers</i> (Ch 4 in BST)	<p><b>Task-Based Lesson</b></p> <p><b>Checkpoint:</b> Coach-Mentee Coaching Session. Bring completed lesson (at least parts 1-5)</p>
9	10/26 James Wood Middle School Rm. 106	Connecting Multiplication to Division	<i>Connecting Division to Multiplication</i> Ch 4 in Constructing Multiplication and Division (Fosnot & Dolk)	<p><b>Journal #4:</b> Strategies and Algorithms</p> <p>Focus Question from Ch 4 in BST</p>
10	11/02 James Wood Middle School Rm. 106	Modeling to Make Meaning of Operations	<p><i>Developing Mathematical Models</i> Ch 5 in Constructing Multiplication and Division (Fosnot &amp; Dolk)</p> <p><i>Beyond Cookies: Understanding Various Division Models</i></p>	<p><b>QUIZ #2</b></p> <p>Multiplication and Division Problem Set</p>
11	11/09 James Wood Middle School Rm. 106	Connecting to Algorithms	<p><i>Algorithms Versus Number Sense</i> Ch 6 in Constructing Multiplication and Division (Fosnot &amp; Dolk)</p> <p>Select one of the following: <i>Interpreting Division Algorithm</i></p>	<p><b>Case Study Due</b></p> <p>Modeling Problem Set</p> <p>Bring discussion talking points for the article you chose to read.</p>

			– BB <i>Students Can Take Us Off Guard-BB</i>	
12	11/16 James Wood Middle School Rm. 106	Decimal PV and Operations	<i>Place Value Representation of Values Less than 1</i> (Ch 6 in BST)  <i>Number and Operations in Base Ten, K–5</i> (Site may not be accessible through school servers.) <a href="http://commoncoretools.me/wp-content/uploads/2015/03/ccss_progression_nbp_k5_2015_03_16.pdf">http://commoncoretools.me/wp-content/uploads/2015/03/ccss_progression_nbp_k5_2015_03_16.pdf</a>	<b>Journal Entry #5</b> Algorithm Problem Set  Make four observations about the progression of mathematical ideas throughout grades K – 5, as stated in the Progressions document. Be prepared to share in class.
13	11/23	<b>NO CLASS</b>	<b>THANKSGIVING</b>	
	11/30 James Wood Middle School Rm. 106	Estimation Number Theory	Computational Estimation-article Mental Arithmetic - article	<b>Task-Based Lesson</b> <b>Checkpoint:</b> Part a, b, c, d, e, and f should be completed and submitted to Blackboard.  Focus Question from Ch 6 in BST
14	12/07 James Wood Middle School Rm. 106	Quantitative Relationships Real Number System Group Work	Real Numbers article - BB	Problem Set
15	12/14 James Wood Middle School Rm. 106	Final Exam Review Presentations	<b>FINAL EXAM</b>	

Note: Faculty reserves the right to alter the schedule as necessary, with notification to students.

## Assessment Rubric(s)

### Numbers Content/Practices Reflection Log Rubric

	#1 C.1.1	#2 C.1.2	#3 C.1.3	#4 C.1.4
NCTM CAEP Element 1A.1: Demonstrate knowledge of major concepts, algorithms, and procedures within and among mathematical domains.				
NCTM CAEP Element 1A.2: Apply knowledge of major concepts, algorithms, procedures, applications in varied contexts, and connections within and among mathematical domains.				
NCTM CAEP Element 1A.3: Explain how concepts, algorithms, procedures, and applications have developed.				
NCTM CAEP Element 1A.4: Apply conceptual and procedural knowledge of major concepts, algorithms, and applications in building new knowledge from prior knowledge and experiences.				
NCTM CAEP Element 2A.1: Use problem solving to develop conceptual understanding and to formulate and test generalizations.				
NCTM CAEP Element 2A.2: Make sense of a wide variety of problems and persevere in solving them.				
NCTM CAEP Element 2A.3: Apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts.				
NCTM CAEP Element 2A.4: Formulate and test conjectures in order to frame generalizations.				
NCTM CAEP Element 2A.5: Monitor and reflect on the process of mathematical problem solving.				
NCTM CAEP Element 2B.1: Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others.				
NCTM CAEP Element 2B.2: Represent and model generalizations using mathematics.				
NCTM CAEP Element 2B.3: Recognize structure and express regularity in patterns of mathematical reasoning.				
NCTM CAEP Element 2B.4: Use multiple representations to model and describe mathematics.				
NCTM CAEP Element 2B.5: Use appropriate mathematical vocabulary and symbols to communicate mathematical ideas				

to others.				
NCTM CAEP Element 2B.6: Demonstrate an appreciation for mathematical rigor and inquiry.				
NCTM CAEP Element 2C.1: Formulate, represent, analyze, interpret, and validate mathematical models derived from real-world contexts or mathematical problems.				
NCTM CAEP Element 2C.2: Demonstrate flexibility in mathematical modeling when confronted with different purposes or contexts.				
NCTM CAEP Element 2F.1: Model how the development of mathematical understanding within and among mathematical domains intersects with the mathematical practices of problem solving, reasoning, communicating, connecting, and representing.				
NCTM CAEP Element 2F.2: Reflect on how the mathematical practices of problem solving, reasoning, communicating, connecting, and representing impact mathematical understanding.				

### Mathematics Historical Context Reflection Rubric

Levels/Criteria	4	3	2	1
	<b>Exceeds Expectations</b>	<b>Meets Expectations</b>	<b>Developing</b>	<b>Does Not Meet Expectations</b>
NCTM Indicator C.1.5 Historical development of number, operations, number systems, and quantity.	Essay describes the historical development of number, operations, number systems, and quantity in depth and provides specific examples.	Essay describes the historical development of number, operations, number systems, and quantity and provides specific examples.	Essay describes the historical development of number, operations, number systems, and quantity and provides an example.	Essay includes incomplete description of historical development of number, operations, number systems, and quantity.
NCTM Indicator C.1.5 Historical perspectives of number, operations, number systems, and quantity.	Essay describes the historical perspectives of number, operations, number systems, and quantity in depth and provides specific examples.	Essay describes the historical perspectives of number, operations, number systems, and quantity and provides specific examples.	Essay describes the historical perspectives of number, operations, number systems, and quantity and provides an	Essay includes incomplete description of historical perspectives of number, operations, number systems, and quantity.

			example.	
NCTM Indicator C.1.5 Contributions of historically significant figures and diverse cultures.	Essay describes contributions of historically significant figures and diverse cultures in depth and provides specific examples.	Essay describes contributions of historically significant figures and diverse cultures and provides specific examples.	Essay describes contributions of historically significant figures and diverse cultures and provides an example.	Essay includes incomplete description of historically significant figures and diverse cultures.

**Rich (Cognitively Demanding) Task and (Lesson Plan) Project Rubric**

<b>Levels/Criteria</b>	<b>4</b>	<b>3</b>	<b>2</b>	<b>1</b>
	<b>Exceeds Expectations</b>	<b>Meets Expectations</b>	<b>Developing</b>	<b>Does Not Meet Expectations</b>
a) CURRICULUM STANDARDS NCTM Indicator 3A.1 Apply knowledge of mathematics curriculum standards for elementary within and across mathematical domains. 3A.2 Relate mathematics curriculum standards to student learning.	Includes the grade level, major concept, objective/goals, VA SOL's, NCTM process standards, mathematical practice CCSS, and prerequisite knowledge.  Cites research on the rationale for choosing a rich task and why your choice in task meets the requirements of a rich task.  Cites research on what elements make a "rich" question or a "high level" question.  Cites research on what is a major mathematical concept or "big mathematical idea".	Includes the grade level, major concept, objective/goals, VA SOL's, NCTM process standards, mathematical practice CCSS, and prerequisite knowledge. Lesson is based on research and it cited correctly.	Includes the grade level, major concept, objective/goals, VA SOL's, NCTM process standards, mathematical practice CCSS, and prerequisite knowledge.	Lesson plan is not based on research. Many aspects of the plan are missing.
b) COMPLETED LESSON PLAN 3C.5 Assist others	Lesson plan provides enough information and	Lesson plan provides enough information and	Lesson plan provides information	Lesson plan is vague – teacher would

<p>in planning lessons and units that incorporate multiple strategies, differentiated instruction for diverse populations, and mathematics-specific and instructional technologies to build all students' conceptual understanding and procedural proficiency.</p>	<p>clarity for a teacher to implement the lesson and task as intended.</p> <p>Assisted peers in developing multiple strategies to build all students' conceptual understanding and procedural proficiency. Assisted peers in identifying diverse populations and modifying the task to build all students' conceptual understanding and procedural proficiency. Assisted peers in aligning mathematics-specific and instructional technologies to build all students' conceptual understanding and procedural proficiency.</p>	<p>clarity for a teacher to implement the lesson and task as intended with some questions.</p> <p>Assisted peers in developing multiple strategies <b>and</b> identifying diverse populations and modifying the task <b>or</b> aligning mathematics-specific and instructional technologies to build all students' conceptual understanding and procedural proficiency.</p>	<p>for a teacher to implement the lesson and task as intended with many questions.</p> <p>Assisted peers in developing multiple strategies <b>or</b> identifying diverse populations and modifying the task <b>or</b> aligning mathematics-specific and instructional technologies to build all students' conceptual understanding and procedural proficiency.</p>	<p>be unable to implement the task as intended.</p> <p>Does not collaborate with peers on lesson plan.</p>
<p>c) QUESTIONS NCTM Indicator 3E.2 Provide instruction that incorporates high quality tasks and a range of questioning strategies. 3E.3 Guide productive mathematical discussions in classrooms centered on key mathematical ideas. 3E.4 Select and apply instructional techniques that assist in identifying and addressing student</p>	<p>Plan contains key questions and student expectations to aid teacher when implementing the task to maintain rigor. Plan contains questions and expected student expectations to address misconceptions. Plan contains questioning strategies to guide productive mathematical discussions in classrooms centered on key mathematical ideas. Questions engage</p>	<p>Plan contains key questions and student expectations to aid teacher when implementing the task to maintain rigor but lacks some clarity. Plan contains questions and expected student expectations to address misconceptions. Questions engage students and teachers in communicating about mathematics.</p>	<p>Plan contains questions low in cognitive demand. Student expectations are also low in rigor.</p>	<p>Plan does not contain key questions. Student expectations are missing.</p>

<p>misconceptions. 3E.5 Engage students and teachers in communicating about mathematics. 3E.6 Use students' misconceptions as opportunities for learning.</p>	<p>students and teachers in communicating about mathematics. Plan describes how student misconceptions will be used as opportunities for learning.</p>			
<p>d) STRATEGIES &amp; MISCONCEPTIONS NCTM Indicator 3C.1 Plan lessons and units that incorporate a variety of strategies. 3C.4 Build all students' conceptual understanding and procedural proficiency in planned lessons and units. 3C.6 Include in planned lessons and units multiple opportunities and solution avenues for students to demonstrate conceptual understanding and procedural proficiency.</p>	<p>Plan contains at least five different strategies that show multiple opportunities and solution avenues for students to demonstrate conceptual understanding and procedural proficiency. Plan contains at least two different misconceptions. Plan describes the connections between the different strategies/misconceptions using descriptions such as similarities, differences, efficiency, visual clarity, mathematical accuracy and/or precision to support students' conceptual understanding and procedural proficiency.</p>	<p>Plan contains four different strategies that show multiple opportunities and solution avenues for students to demonstrate conceptual understanding and procedural proficiency. Plan contains one misconception. Plan describes the connections between the different strategies/misconceptions to support students' conceptual understanding and procedural proficiency.</p>	<p>Plan contains three different strategies that show multiple opportunities and solution avenues for students to demonstrate conceptual understanding and procedural proficiency. Description of conceptual understanding and procedural proficiency is incomplete.</p>	<p>Student strategies and misconceptions lack a complete listing and in-depth understanding.</p>
<p>e1) TECHNOLOGY NCTM Indicator 3C.1 Include mathematics-specific and instructional technologies in planned lessons and units.</p>	<p>Your choice of technology is explained regarding how it is math-specific and supports the task. The tool is specific to the task (ie: the geoboard on NLVM, and not simply "iPads"). Links to the web or</p>	<p>Your choice of technology is explained regarding how it is math-specific and supports the task. The tool is specific to the task (ie: the geoboard on NLVM, and not simply "iPads").</p>	<p>Your choice of technology is not explained regarding how it is math-specific and supports the task <b>or</b> the tool is not specific.</p>	<p>Your choice of technology is not explained regarding how it is math-specific and supports the task <b>and</b> the tool is not specific.</p>

	appstore are provided and screen captures of the tool are included.			
e2) Developmental TECHNOLOGY NCTM Indicator 5B.3 Engage students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge.	Your choice of technology is explained regarding how it will enhance learning. Tool engages students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge. Plan contains a detailed explanation of how the students will interact with the tool.	Your choice of technology is explained regarding how it will enhance learning. Tool engages students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge.	Your choice of technology is not explained regarding how it will enhance learning <b>or</b> the tool does not engage students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge.	Your choice of technology is not explained regarding how it will enhance learning <b>and</b> the tool does not engage students in developmentally appropriate mathematical activities and investigations that include mathematics-specific technology in building new knowledge.
f1) DIFFERENTIATION NCTM Indicator 3A.3 Demonstrate how mathematics curriculum standards and learning progressions impact the teaching of elementary students at different developmental levels and coaching/mentoring elementary classroom teachers.	Plan identifies the grade level standard (VA SOL & CCSS) and at least two other grade levels and describes the progression and vertical alignment. Modifications to the lesson are given for different developmental levels to meet all student needs. Collaborated with peers to coach and give feedback on the differentiation of others' numbers task.	Plan includes how to modify the lesson to gear down and gear up to meet all student needs but is lacking clarity <b>or</b> completeness. Modifications are given that are appropriate for the given level but may lack clarity <b>or</b> completeness. Collaborated with peers to coach and give feedback on the differentiation of others' numbers task.	Plan includes how to modify the lesson to gear down and gear up to meet all student needs but is lacking clarity <b>and</b> completeness . Grade modifications are given that are appropriate for the given level but may lack clarity <b>and</b> completeness .	Plan differentiation and modifications are very minimal.
f2) DIVERSE POPULATIONS 3C.2 Plan lessons and units addressing student	Modifications to the lesson are given for diverse populations to meet all student needs.	Modifications are given for diverse populations but may lack clarity <b>or</b> completeness.	Modifications are given for diverse populations but may lack	Plan modifications are very minimal.

differences and diverse populations and how these differences influence student learning of mathematics.	Explanation of how student differences may influence their learning of mathematics.		clarity <b>and</b> completeness	
g) HANDOUT & KEY <u>Numbers Task:</u> handout is clear and applicable for the grade level. Answer key is provided.	Numbers Task handout is clear and applicable for the grade level. Answer key is provided.	Numbers Task handout is applicable for the grade level but lacks clarity. Answer key is provided.	Numbers Task handout is applicable for the grade level but lacks clarity. Answer key is not provided.	Numbers Task handout is not applicable for the grade level. Answer key is not provided.
h) TASK NCTM Indicator 3E.1 Implement and promote techniques for actively engaging students in learning and doing mathematics.	Numbers task implements and promotes techniques for actively engaging students in learning and doing mathematics.	Numbers Task_has a cognitive demand of "Procedures with Connections"	Numbers Task_has a cognitive demand of "Procedures without Connections"	Numbers Task_has a cognitive demand of "Memorization"
i) LESSON REFLECTION - JUSTIFICATION NCTM Indicator 5B.1 Engage students in developmentally appropriate mathematical activities and investigations that require active engagement in building new knowledge.	Reflection shows evidence that students were engaged in developmentally appropriate mathematical activities and investigations that require active engagement in building new knowledge.	Reflection shows evidence that students were engaged in investigations that require active engagement in building new knowledge.	Reflection shows evidence that students were engaged in investigations that require active engagement.	Reflection of lesson implementation is missing.
j) LESSON REFLECTION - PROBLEM SOLVING 5B.4 Facilitate students' ability to develop future inquiries based on current analyses.	Reflection shows evidence that you facilitated students' ability to develop future inquiries based on current analyses.	Reflection shows evidence that you facilitated students' ability to develop inquiries about the task.	Reflection shows evidence that you facilitated students' ability solve the task.	There is no expectation of students communicating their problem solving strategies.
k) COACHING/ASSISTING	Assist peers in designing a task that uses	Assist peers in designing a task that is developmentally	Assist peers in thinking about their	Does not collaborate with peers

<p>NCTM INDICATOR 5B.2 Coach/mentor teachers in using developmentally appropriate mathematical activities and investigations that require active student engagement in building new knowledge.</p>	<p>developmentally appropriate mathematical activities and investigations that require active student engagement in building new knowledge.</p>	<p>appropriate and uses investigations.</p>	<p>task.</p>	<p>during task creation.</p>
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